**Can all foods be classified as a salad, a soup or a sandwich?**

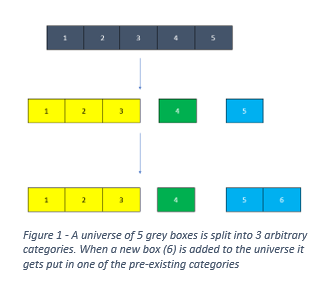
# **ABSTRACT**

The main body of work that goes into classification and categorisation is defining the model that gives the rules of organising and determines which classes elements get placed into. The main goal of this paper is to present a new method of modular model based classification, building on the framework and ideas set out in *“Ordination on the Basis of Fuzzy Set Theory”* specifically in the domain of ***complete universe categorisation***. The author defines a new technique of determining objects’ 3-dimensional structural similarity (herein described as ***3-dimensional object concave hull intersection estimation*** or for the sake of brevity, ***3D concave hull estimation***), heavily inspired by the work in *“Cognition and Categorization”* as well as influenced by “*Learning visual biases from human imagination”*. Additionally, the author defines the class structure as a concept before going on to analyse individual segments of the question and the properties of the model. Finally, the author employs a method of model analysis formulated using the normal equation for linear regression as a basis. Ultimately the author determines that… FINDINGS GO HERE

# **INTRODUCTION**

Given an appropriate model, anything can be categorised into any arbitrary group. A common misconception in the field of classification theory is that categorising objects is about choosing the correct labels. Whilst this can be important, the main body of work that goes into classification and categorisation is defining the model that gives the rules of organising and determines which classes elements get placed into. A model can be interpreted as a child’s toy, where different 3D shapes can be pushed through corresponding holes: The actual shapes used (or in our case, the elements being sorted) are irrelevant, but the concept of having an entity that contains not only all the different classes - but the restraints as well, acts as a good analogy for a model. Models contain all the class names, the rules that determine which objects get put into what classes and the variables/properties that every item holds[[1]](#footnote-1)

The main goal of this paper is to present a new method of classification, building on the framework and ideas set out in *“Ordination on the Basis of Fuzzy Set Theory”* [*(****Roberts David, 1986***)](#_Roberts,_D.W._(1986).) specifically in the domain of ***complete universe categorisation*** (*figure 1*). The author defines a new technique of determining objects’ 3-dimensional structural similarity (herein described as ***3 dimensional object concave hull intersection estimation*** or for the sake of brevity, ***3D concave hull estimation***), heavily inspired by the work in *“Cognition and Categorization”* [***(Rosch et al, 1980)***](#_Culbert,_S.S.,_Rosch,) as well as influenced by “*Learning visual biases from human imagination”* [***(Oliva et al, 2015)***](#_Vondrick,_C.,_Pirsiavash,).

Additionally, the author defines the other measures of similarity employed as well as the class structure as a concept before going on to analyse individual segments of the question and the properties of the model.

Finally, the author employs a method of model analysis formulated using the normal equation for linear regression as a basis [***(Uyanık & Güler, 2013)***](#_Uyanık,_G.K._and).

**Part I: Research review**

Over the course of my project, I have looked at sources and academic papers in a broad range of topic areas to reinforce and solidify my understanding of my topic. This research review attempts to analyse and evaluate those texts to assess their meaning, impact and validity in the field of classification and categorisation. To aid this examination, I have split my sources into 6 distinct topic groups and will be evaluating each, one at a time. These topic headings are as follows: [**Logic**](#_Logic), [**Set theory**](#_Set_theory), [**Group theory**](#_Group_theory), [**Information systems**](#_Information_systems), [**Classification theory**](#_Classification_theory) and [**Miscellaneous**](#_Miscellaneous) (for sources that didn’t fit into any of the above)

# ***Logic***

The smallest body of research I did was looking at concepts in mathematical logic. I looked at this area because I wanted to gain a better understanding of mathematical models as well as how models can be used to simulate complex concepts in the real world. This was important for me to learn about because in my dissertation I chose to use maths heavily in an area in which it has rarely been employed in the past. The few crossover papers I found (for example:[***Roberts David, 1986***),](#_Roberts,_D.W._(1986).) fall more neatly into another category so I will not discuss them in this section.

In “*The Inescapability of Gettier problems”* ([***Zagzebski Linda, 1994***](#_Zagzebski,_L._(1994).)), Zagzebski describes the problem of perceived truths (either through evidence or extrapolation) being false: a problem that affects models of all kinds, but especially those in classification theory. This was helpful for me in understanding some of the limitations that my model would have. As models are, by tautology, designed to depict the real world, this source was useful to me despite being written by a philosopher: looking at life in general as opposed to mathematics specifically.

The ideas Zagzebski introduces in this paper are also raised in a more general way in [***Philip Welch and Benedikt Löwe’s 2001 paper***](#_Löwe,_B._and)***:*** “*Set-Theoretic Absoluteness and the Revision Theory of Truth”*. Whilst most of this paper was either too advanced mathematically - or irrelevant to my topic – it does bring up some interesting points about the nature of proof. Welch and Löwe argue that it is always easier to disprove something than it is to cover all cases and prove it is true. In the field of classification theory, having a model that accurately reflects the real world is one of the fundamental requirements of any classification (if the model is incorrect then any analysis done on data produced by the system will be inherently wrong). Both papers highlight some important truths about all models: It is an incredibly difficult task to determine whether or not the model is reflective of life or whether it is just true for the data that it is tested on. Additionally, it is far easier to find a sample unit for which the model breaks down than it is to verify that the model holds true *omnis*. Succinctly, it is hard to tell if the model works or just “looks” like it works.

In addition to these two main sources that I looked at, I also explored other papers including “*Mathematical Logic as based on the theory of types”*[*(****Russel Bertrand, 1908****)*](#_Russel,_B._(1908)._1)*.* However, I found this paper was too mathematically advanced for me to fully grasp and it didn’t give me as much insight into the topic as I would have liked.

I found that the paper “*Knowledge and certainty*” *(*[***Stanley Jason, 2008***](#_Stanley,_J._(2008).)*),* was perfect for summing up my findings in the area of mathematical logic. Stanley describes how knowledge needs 3 things to be considered true (or, in my case, how a model requires 3 things to be properly representative): Truth, belief and certainty. The first of these is self-explanatory – for the results of a model to be considered a statement of knowledge, they must have the appearance of recognising integral (and observable) aspects of real life. This means that the data in your model has to be applicable in the most basic of situations, otherwise it won’t be the right fit for more complex cases.

The second requirement is harder to describe in the case of mathematical models (a drawback of this paper was its conceptual framing). In this instance, I am interpreting it to be the same as holding a hypothesis. This is the idea that once you have a model that is true for some arbitrary data points, then there must be the belief that that model holds for all items in the universe (since in many situations it is often impractical or impossible to check every unit).

Finally, Stanley argues that an element of certainty is necessary for solidifying findings as facts. In the case of me and my model, certainty is determined as the confidence in the hypothesis created in the above paragraph. It is symbolic of to what extent we are confident in our belief that the model is true for all elements. It is here that problems from the earlier discussions of Zagzebski and Welch & Löwe ’s papers: Even if we have a model and a hypothesis that it holds for everything, we can never reach full certainty that this model doesn’t suffer from Zagzebski’s “Gettier problems” ([***Zagzebski Linda, 1994***](#_Zagzebski,_L._(1994).)), and that a complete demonstration - which would eliminate this possibility – probably cannot be achieved ([***Löwe and Welch, 2001***](#_Löwe,_B._and))

# ***Set theory***

I looked at key concepts of set theory to reinforce my pre-existing knowledge in this area and to look at examples of one of the most basic class structures. I was particularly interested in “fuzzy” set theory – a variant of the standard structure where the degree of class membership is represented by a number – and decided to implement it in my method.

Other than some basic introductory level texts that I looked at to consolidate my understanding of this area, the main paper that really aided my research (and influenced my dissertation) was *“Ordination on the basis of fuzzy set theory”* [(***Roberts David, 1986***)](#_Roberts,_D.W._(1986).). I will touch upon this paper again in the “Classification theory” section of this document, but the main scope of this paper will be discussed here. This paper was especially helpful to me, as it was written specifically for the purposes of classification theory and was one of the few sources that I found which used set theory to accomplish this goal. The beginning of the paper serves as an introduction - or a reminder – to the world of set theory and the nuances with fuzzy sets. Later in the text, Roberts uses this foundation to define a simplistic but efficient system of classification of which my project is a subtype. Most of the core concepts employed in my dissertation are techniques that Roberts used: adapted to be simpler, more adaptable and thus by extension, more generalised.[[2]](#footnote-2) Roberts also produced a follow up paper clarifying some points of confusion and extending his original text [***(Roberts David, 1987)***](#_Roberts,_D.W._(1987).)which I will analyse in another section.

I chose sets as the main data structure for my project partially because it is an area that I was already fairly well acquainted with, but also because it provided me a flexible structure that is easily adaptable and intuitive for all people to understand.

Set theory is a key part of my project but discussing it fully here is beyond the breadth of this research review. Despite being a relatively new area of mathematics, there are lots of great resources to aid in learning the fundamentals. I will go over the core concepts of fuzzy set theory briefly in my dissertation but this itself requires at least a GCSE-equivalent understanding of the basics so some pre-reading may be required (Just the first page of “<https://www.bbc.co.uk/bitesize/guides/z8nfrdm/revision/1>” would be entirely sufficient)

# ***Group theory***

I explored this area as a more complex, higher-level structure than the basic set. I also needed to build up my knowledge in this area for to help me better understand and implement some of the information systems that I looked at. Initially, I wanted groups to be the main abstract class structure that I used in my dissertation: However, as I proceeded with my project, I decided that groups weren’t the best fit and I would rarely use them. This was because the nature of groups doesn’t combine well with edge/boundary cases and membership to a group is definite – unlike the fuzzy sets I also looked at. Ultimately, I decided that these factors were too significant for me to rely upon group theory as an integral area of my dissertation and I gradually stopped looking at it.

Most of the texts that I looked at – for example, *“Easy Group Theory”* [***(Miller George, 1992)***](#_Miller,_G.A._(1922).) and *“What Are the Fundamental Concepts of Group Theory?”*[***(Burn Bob, 1996)***](#_Burn,_B._(1996).) *–* were introductory level texts to help me understand the core components of group theory. Whilst they were useful in this regard - helping me build my understanding - I found that once I started moving away from the idea of using groups, they quickly became redundant pieces of information. Additionally, because groups have been rarely used in classification models in the past (and after researching it has become evident to me why), it was difficult to find papers that addressed the specific area that I wanted them to cover. This meant that a majority of the papers or journals were full of information that either didn’t help me or didn’t apply to my topic.

Another issue I had was that group theory is a very young concept mathematically: especially in comparison to the information systems and theory papers I was also reading. This means that due to various technological advances (such as mainstream computer usage as well as the desire for artificial intelligence in the 80s and 90s), methods have changed a lot in the time between the two types of paper being produced. Group theory is a largely undiscovered field that has rapidly changing innovations from [***Mackey George, 1973***](#_Mackey,_G.W._(1973).)up to the present day. On the other hand, classification theory has only very recently been re-explored after years of uninterest (largely caused by the fact that the systems we developed long ago still work very effectively)[[3]](#footnote-3)

Overall, group theory was a very interesting topic area for me to explore and built up my knowledge (in set theory as well), even though it ultimately wasn’t the right fit for my project.

# ***Information systems***

Most of my early research time was placed into exploring pre-existing information systems to look at how they work and how effective they were. This area of research is essentially applied classification theory, so there is a large overlap between the theory and the implementations that I looked at. I studied some methods of data retrieval and how they worked - which was very helpful in guiding me in creating my own system. However, the largest portion of papers on this topic are written by bibliographers (with the specific intention of analysing books and library storage) so whilst there is a relation, I tended to get more conceptual ideas out of some of these applied papers (for example: [***Farradane Jason, 1950***](#_Farradane,_J.E.L._(1950).).)

# ***Classification theory***

I found most of these papers after I had done a large portion of my initial research, when I was finally in a position to understand them. These papers were influential in the decisions that I made in my dissertation, and I often adapted ideas that I read about to make my model.

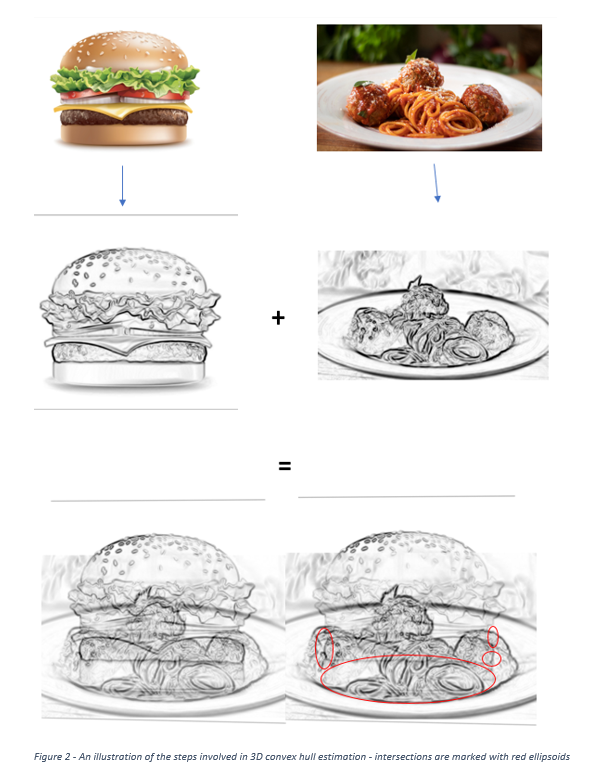
# ***Miscellaneous***

Inside this bracket I have collected a bunch of papers and other resources that helped me in developing ideas for my dissertation. Most of these only outlined formulae or methods and so couldn’t really be placed into any of the above categories.

# **Part II: Dissertation**

Can all foods be classified as a salad, a soup or a sandwich?

In short, the answer is yes. Whether or not that would be a good model is debatable, but in essence - any number of objects could always be put into groups and given arbitrary labels. It is part of the very nature of an entity – a French “pomme” is still an English “apple” despite having different labels



# **Glossary**

1. **3-dimensional object concave hull intersection estimation –** A method of estimating the structural similarity between two intangible 3D objects by measuring or estimating the proportion of crossover in their relative surface areas (figure 2)
2. **Complete universe categorization –** The whole of a dynamically growing universe is classified into a finite number or arbitrary groups
3. **Universe –** Every element in the group of objects being sorted

# **Bibliography**

# Bencivenga, E. (1976). Set Theory and Free Logic. *Journal of Philosophical Logic*, [online] 5(1), pp.1–15. Available at: <https://www.jstor.org/stable/30226131> [Accessed 15 Nov. 2022].

# Berlin, I. and Hardy, H. (1996). *Concepts and Categories: Philosophical Essays*. REV - Revised, 2 ed. [online] *JSTOR*. Princeton University Press. Available at: [https://www.jstor.org/stable/j.ctt46n3tc](https://www.jstor.org/stable/j.ctt46n3tc%20) [Accessed 27 Feb. 2023].

# Borel, A. and Harish-Chandra (1962). Arithmetic Subgroups of Algebraic Groups. *Annals of Mathematics*, [online] 75(3), pp.485–535. doi:<https://doi.org/10.2307/1970210>.

# Bunyamin, H. (n.d.). *Deriving Normal Equation of Linear Regression Model*. [online] hbunyamin.github.io. Available at: <https://hbunyamin.github.io/machine-learning/Normal_equation/> [Accessed 25 Feb. 2023].

# Burn, B. (1996). What Are the Fundamental Concepts of Group Theory? *Educational Studies in Mathematics*, [online] 31(4), pp.371–377. Available at: [https://www.jstor.org/stable/3482970](https://www.jstor.org/stable/3482970%20) [Accessed 17 Nov. 2022].

# Culbert, S.S., Rosch, E. and Lloyd, B.B. (1980). Cognition and Categorization. *The Modern Language Journal*, [online] 64(2), p.284. doi:<https://doi.org/10.2307/325356>.

# Farradane, J.E.L. (1950). A SCIENTIFIC THEORY OF CLASSIFICATION AND INDEXING AND ITS PRACTICAL APPLICATIONS. *Journal of Documentation*, 6(2), pp.83–99. doi:<https://doi.org/10.1108/eb026155>.

# Farradane, J.E.L. (1952). A SCIENTIFIC THEORY OF CLASSIFICATION AND INDEXING: FURTHER CONSIDERATIONS. *Journal of Documentation*, 8(2), pp.73–92. doi:<https://doi.org/10.1108/eb026182>.

# Gottwald, S. (2006). Universes of Fuzzy Sets and Axiomatizations of Fuzzy Set Theory. Part II: Category Theoretic Approaches. *Studia Logica: An International Journal for Symbolic Logic*, [online] 84(1), pp.23–50. Available at: [https://www.jstor.org/stable/20016819](https://www.jstor.org/stable/20016819%20) [Accessed 15 Nov. 2022].

# Hamlyn, D.W. (1959). Categories, Formal Concepts and Metaphysics. *Philosophy*, [online] 34(129), pp.111–124. Available at: [https://www.jstor.org/stable/3748729](https://www.jstor.org/stable/3748729%20) [Accessed 14 Nov. 2022].

# Jimenez, S., Becerra, C., Gelbukh Cic-Ipn, A., Dios Bátiz, A. and Mendizábal (2013). SOFTCARDINALITY-CORE: Improving Text Overlap with Distributional Measures for Semantic Textual Similarity. [online] 1, pp.194–201. Available at: <https://aclanthology.org/S13-1028.pdf> [Accessed 14 Mar. 2023].

# Lee, H.N. (1931). THE MEANING OF THE NOTATION OF MATHEMATICS AND LOGIC. *The Monist*, [online] 41(4), pp.594–617. Available at: [https://www.jstor.org/stable/27901326](https://www.jstor.org/stable/27901326%20) [Accessed 22 Nov. 2022].

# Löwe, B. and Welch, P.D. (2001). Set-Theoretic Absoluteness and the Revision Theory of Truth. *Studia Logica: An International Journal for Symbolic Logic*, [online] 68(1), pp.21–41. Available at: [https://www.jstor.org/stable/20016296](https://www.jstor.org/stable/20016296%20) [Accessed 8 Nov. 2022].

# Mackey, G.W. (1973). Group Theory and Its Significance for Mathematics and Physics. *Proceedings of the American Philosophical Society*, [online] 117(5), pp.374–380. Available at: [https://www.jstor.org/stable/986606](https://www.jstor.org/stable/986606%20) [Accessed 17 Nov. 2022].

# Malmquist, G. (2022). *Salad Theory*. [online] <https://saladtheory.github.io>. [Accessed 10 Nov. 2022].

# Miller, G.A. (1922). Easy Group Theory. *The Scientific Monthly*, [online] 15(6), pp.512–519. Available at: [https://www.jstor.org/stable/6660](https://www.jstor.org/stable/6660%20) [Accessed 17 Nov. 2022].

# Parsons, J. (1996). An Information Model Based on Classification Theory. *Management Science*, [online] 42(10), pp.1437–1453. Available at: [https://www.jstor.org/stable/2634376](https://www.jstor.org/stable/2634376%20) [Accessed 8 Nov. 2022].

# Roberts, D.W. (1986). Ordination on the Basis of Fuzzy Set Theory. *Vegetatio*, [online] 66(3), pp.123–131. Available at: [https://www.jstor.org/stable/20037322](https://www.jstor.org/stable/20037322%20) [Accessed 15 Nov. 2022].

# Roberts, D.W. (1987). An anticommutative difference operator for fuzzy sets and relations. *Fuzzy Sets and Systems*, 21(1), pp.35–42. doi:<https://doi.org/10.1016/0165-0114(87)90150-3>.

# Rosch, E., Mervis, C.B., Gray, W.D., Johnson, D.M. and Boyes-Braem, P. (1976). Basic objects in natural categories. *Cognitive Psychology*, 8(3), pp.382–439. doi:<https://doi.org/10.1016/0010-0285(76)90013-x>.

# Rosch, E.H. (1973). Natural categories. *Cognitive Psychology*, 4(3), pp.328–350. doi:<https://doi.org/10.1016/0010-0285(73)90017-0>.

# Ross, B.H. and Spalding, T.L. (1994). Concepts and Categories. *Thinking and Problem Solving*, pp.119–148. doi:<https://doi.org/10.1016/b978-0-08-057299-4.50010-4>.

# Russel, B. (1908). Mathematical Logic as Based on the Theory of Types. *American Journal of Mathematics*, [online] 30(3), pp.222–262. doi:<https://www.jstor.org/stable/2369948>.

# Stanley, J. (2008). Knowledge and Certainty. *Philosophical Issues*, [online] 18, pp.35–57. Available at: [https://www.jstor.org/stable/27749898](https://www.jstor.org/stable/27749898%20) [Accessed 14 Nov. 2022].

# Stein, N. (2016). Causes and Categories. *Noûs*, [online] 50(3), pp.465–489. Available at: [https://www.jstor.org/stable/26631401](https://www.jstor.org/stable/26631401%20) [Accessed 14 Nov. 2022].

# Uyanık, G.K. and Güler, N. (2013). A Study on Multiple Linear Regression Analysis. *Procedia - Social and Behavioral Sciences*, 106(1), pp.234–240. doi:<https://doi.org/10.1016/j.sbspro.2013.12.027>.

# Vickery, B.C. (1953). SYSTEMATIC SUBJECT INDEXING. *Journal of Documentation*, 9(1), pp.48–57. doi:<https://doi.org/10.1108/eb026190>.

# Vickery, B.C. (1955). DEVELOPMENTS IN SUBJECT INDEXING. *Journal of Documentation*, 11(1), pp.1–11. doi:https://doi.org/10.1108/eb026209.

# Vickery, B.C. (1963). SCIENTIFIC INFORMATION: PROBLEMS AND PROSPECTS. *Minerva*, [online] 2(1), pp.21–38. Available at: [https://www.jstor.org/stable/41821596](https://www.jstor.org/stable/41821596%20) [Accessed 29 Nov. 2022].

# Vondrick, C., Pirsiavash, H., Oliva, A. and Torralba, A. (2015). *Learning visual biases from human imagination*. [online] Available at: [https://proceedings.neurips.cc/paper/2015/file/8f53295a73878494e9bc8dd6c3c7104f-Paper.pdf](https://proceedings.neurips.cc/paper/2015/file/8f53295a73878494e9bc8dd6c3c7104f-Paper.pdf%20) [Accessed 2 Mar. 2023].

# Zagzebski, L. (1994). The Inescapability of Gettier Problems. *The Philosophical Quarterly*, 44(174), p.65. doi:<https://doi.org/10.2307/2220147>.

1. Note that throughout this paper, the author uses the words item, entity, element and object interchangeably although strictly speaking, they should be split into two: entities, objects, articles, etc have properties (as well as relations - [***Farradane Jason, 1950***](#_Farradane,_J.E.L._(1950).)), Whereas items and elements are objects that belong to collections. [↑](#footnote-ref-1)
2. Note that this isn’t necessarily always going to be a good thing. It means that whilst the framework I end up providing is extremely modular, it isn’t always going to be ready “out of the box” and it is up to the individual user to modify to suit their needs. [↑](#footnote-ref-2)
3. The rise in neural networks and computer vision has spawned much more interest in the field of classification theory as we attempt to make machines that can do complex scene recognition tasks (such as in [***Olivia et al, 2015***](#_Vondrick,_C.,_Pirsiavash,)). [↑](#footnote-ref-3)